

Financial Review Online Appendix for “Trade Signing in Fast Markets”
Allen Carrion, Madhuparna Kolay
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A: Sample Stock Characteristics

Table A.1

Sample stocks

Sample was selected for NASDAQ by Terrence Hendershott and Ryan Riordan. Sample period is January 2008 – December 2009 and February 22, 2010 – February 26, 2010. Listing venue, price and market capitalization are from CRSP as of February 26, 2010. Dollar trading volumes are from TAQ for trades between 9:30 am and 4:00:00 pm and are averaged over all days in sample for each name. Stocks are sorted in descending order by market capitalization.

Ticker	Name	Listing	Mkt cap (billions)	Price	Avg. Dollar Trading Volume		
					Nasdaq	CT	NASDAQ Share
AAPL	APPLE INC	NASDAQ	185.548	204.62	1,654.244	4,229.780	38.2%
PG	PROCTER & GAMBLE CO	NYSE	183.803	63.28	186.570	889.604	20.7%
GE	GENERAL ELECTRIC	NYSE	171.357	16.06	350.167	1,710.448	19.4%
PFE	PFIZER INC	NYSE	141.635	17.55	190.253	908.159	20.4%
CSCO	CISCO SYSTEMS INC	NASDAQ	139.305	24.33	544.225	1,216.373	43.7%
GOOG	GOOGLE INC	NASDAQ	128.612	526.80	976.411	2,197.014	43.5%
HPQ	HEWLETT PACKARD	NYSE	119.564	50.79	168.456	762.414	21.6%
INTC	INTEL CORP	NASDAQ	113.408	20.53	557.855	1,285.776	42.6%
DIS	DISNEY WALT CO	NYSE	60.590	31.24	81.993	377.655	20.9%
MMM	3M CO	NYSE	57.045	80.15	65.307	329.624	19.3%
AMGN	AMGEN INC	NASDAQ	55.438	56.61	243.441	532.812	45.6%
AMZN	AMAZON COM INC	NASDAQ	52.715	118.40	306.855	727.308	42.9%
AXP	AMERICAN EXPRESS CO	NYSE	45.703	38.19	111.628	493.304	22.1%
GILD	GILEAD SCIENCES	NASDAQ	42.952	47.61	200.488	435.583	45.7%
CMCSA	COMCAST CORP	NASDAQ	33.884	16.44	192.761	417.744	45.8%
DOW	DOW CHEMICAL CO	NYSE	32.565	28.31	60.744	293.240	20.6%
HON	HONEYWELL INTERNATIONAL	NYSE	30.704	40.16	54.990	242.639	22.2%
EBAY	EBAY INC	NASDAQ	29.903	23.02	159.904	363.127	43.7%
PNC	P N C FINANCIAL SERVICES GRP INC	NYSE	27.792	53.76	51.977	303.458	17.3%
GLW	CORNING INC	NYSE	27.483	17.63	63.275	276.531	22.1%
CELG	CELGENE CORP	NASDAQ	27.363	59.52	130.188	281.530	45.9%
COST	COSTCO WHOLESALE CORP	NASDAQ	26.850	60.97	144.949	330.337	43.5%
ESRX	EXPRESS SCRIPTS	NASDAQ	26.445	96.01	89.624	193.355	46.2%
MOS	MOSAIC COMPANY	NYSE	25.993	58.39	110.995	519.619	19.5%
DELL	DELL INC	NASDAQ	25.911	13.24	200.338	464.816	41.8%
KMB	KIMBERLY CLARK	NYSE	25.286	60.74	27.274	163.386	16.4%
ADBE	ADOBE SYSTEMS	NASDAQ	18.211	34.65	115.867	245.726	46.5%
AGN	ALLERGAN INC	NYSE	17.763	58.43	23.249	124.567	18.2%
CB	CHUBB CORP	NYSE	16.753	50.46	29.610	168.466	17.5%
AMAT	APPLIED MATERIALS INC	NASDAQ	16.442	12.24	150.360	339.454	43.4%
GENZ	GENZYME CORP	NASDAQ	15.221	57.20	102.936	220.846	46.9%

BIIB	BIOGEN IDEC INC	NASDAQ	15.120	55.01	86.319	193.493	44.7%
BHI	BAKER HUGHES INC	NYSE	14.946	47.92	64.653	275.455	22.8%
GPS	GAP INC	NYSE	14.835	21.50	40.019	170.821	23.6%
SWN	SOUTHWESTERN ENERGY CO	NYSE	14.726	42.55	44.165	214.302	20.7%
KR	KROGER COMPANY	NYSE	14.363	22.10	39.537	191.461	20.4%
CTSH	COGNIZANT TECHNOLOGY SOLS	NASDAQ	14.322	48.13	63.043	145.206	43.7%
BRCM	BROADCOM CORP	NASDAQ	13.737	31.32	128.856	295.870	43.3%
AA	ALCOA INC	NYSE	13.570	13.30	90.068	410.228	21.4%
ISRG	INTUITIVE SURGICAL INC	NASDAQ	13.516	347.14	99.863	220.226	45.3%
CSL	CARLISLE COMPANIES	NYSE	2.064	34.30	2.569	14.552	17.1%
AINV	APOLLO INVESTMENT CORP	NASDAQ	2.055	11.66	8.404	19.069	42.2%
LECO	LINCOLN ELECTRIC HOLDINGS INC	NASDAQ	2.034	47.70	9.476	17.911	52.4%
SFG	STANCORP FINANCIAL GROUP	NYSE	2.030	42.98	2.975	17.587	16.9%
FL	FOOT LOCKER INC	NYSE	2.030	12.97	5.874	30.986	19.2%
ERIE	ERIE INDEMNITY CO	NASDAQ	2.029	39.62	2.739	4.706	57.2%
LSTR	LANDSTAR SYSTEM CONCUR	NASDAQ	2.004	39.89	15.164	31.165	47.6%
CNQR	TECHNOLOGIES INC	NASDAQ	1.943	39.34	12.837	27.586	46.9%
EWBC	EAST WEST BANCORP INC	NASDAQ	1.927	17.52	8.506	19.787	42.2%
JKHY	HENRY JACK & ASSOC INC	NASDAQ	1.908	22.58	8.708	17.120	49.8%
FCN	F T I CONSULTING	NYSE	1.904	36.74	9.878	50.463	20.3%
CBT	CABOT CORP	NYSE	1.899	29.06	2.247	12.376	17.6%
PNY	PIEDMONT NATURAL GAS INC	NYSE	1.895	25.83	2.057	12.067	16.2%
GAS	NICOR INC	NYSE	1.884	41.65	4.612	23.929	18.6%
BRE	B R E PROPERTIES	NYSE	1.860	33.71	5.196	36.006	14.5%
CR	CRANE CO	NYSE	1.855	31.67	2.130	11.512	17.7%
FMER	FIRSTMERIT CORP	NASDAQ	1.838	21.13	10.510	20.940	49.8%
COO	COOPER COMPANIES INC	NYSE	1.832	40.06	3.291	19.065	16.6%
ISIL	INTERSIL CORP	NASDAQ	1.826	14.84	23.484	53.516	43.5%
MELI	MERCADOLIBRE	NASDAQ	1.815	41.14	11.878	28.770	40.0%
ROC	ROCKWOOD HOLDINGS INC	NYSE	1.782	23.99	2.556	13.784	18.5%
CSE	CAPITALSOURCE	NYSE	1.777	5.50	5.377	30.523	15.2%
CHTT	CHATTEM INC	NASDAQ	1.774	93.48	12.617	27.768	46.3%
ARCC	ARES CAPITAL	NASDAQ	1.737	13.07	4.232	9.884	42.6%
CKH	SEACOR HOLDINGS	NYSE	1.727	76.38	3.585	18.195	19.1%
NSR	NEUSTAR INC	NYSE	1.726	23.18	3.201	15.541	19.8%
PTP	PLATINUM UNDERWRITERS HLDGS LTD	NYSE	1.710	37.39	2.958	20.932	14.2%
CPWR	COMPUWARE CORP	NASDAQ	1.703	7.49	11.685	26.184	44.2%
FULT	FULTON FINANCIAL CORP PA	NASDAQ	1.697	9.62	7.652	15.745	47.2%
AYI	ACUITY BRANDS	NYSE	1.692	38.98	4.146	22.083	18.6%
SF	STIFEL FINANCIAL	NYSE	1.689	54.70	2.434	14.160	17.5%

NUS	NU SKIN ENTERPRISES INC	NYSE	1.676	26.72	1.120	6.844	14.9%
BARE	BARE ESCENTUALS	NASDAQ	1.674	18.18	5.483	14.289	36.3%
LPNT	LIFEPOINT HOSPITALS INC	NASDAQ	1.673	30.50	11.012	25.294	44.2%
CRI	CARTERS INC	NYSE	1.664	28.66	3.179	18.992	16.9%
AMED	AMEDISYS INC	NASDAQ	1.630	57.65	16.055	37.936	44.7%
BXS	BANCORPSOUTH	NYSE	1.625	19.47	2.892	17.268	16.3%
LANC	LANCASTER COLONY CORP	NASDAQ	1.623	57.54	3.880	7.474	52.6%
CETV	CENTRAL EUROPEAN MEDIA ENT LTD	NASDAQ	1.513	26.99	10.819	23.362	46.5%
MANT	MANTECH INTERNATIONAL CORP	NASDAQ	1.106	49.38	7.760	15.189	50.8%
NXTM	NXSTAGE MEDICAL	NASDAQ	0.498	10.65	0.449	1.013	43.6%
CTRN	CITI TRENDS INC	NASDAQ	0.438	29.74	2.481	5.538	45.9%
RVI	RETAIL VENTURES	NYSE	0.438	8.94	0.210	1.410	13.4%
MAKO	MAKO SURGICAL	NASDAQ	0.436	13.21	0.283	0.703	37.4%
MOD	MODINE MANUFACTURING	NYSE	0.435	9.40	0.554	2.924	16.3%
ROG	ROGERS CORP	NYSE	0.433	27.45	0.898	4.084	18.8%
KTII	K TRON INTL INC	NASDAQ	0.424	149.46	0.684	1.369	49.8%
KNOL	KNOLOGY INC	NASDAQ	0.421	11.45	0.798	1.723	47.3%
PPD	PRE PAID LEGAL SERVICES INC	NYSE	0.418	41.64	0.633	3.928	14.7%
DCOM	DIME COMMUNITY BANCSHARES	NASDAQ	0.418	12.14	1.679	3.339	49.7%
BW	BRUSH ENGINEERED MATERIALS INC	NYSE	0.416	20.54	1.160	5.631	19.4%
SJW	S J W CORP	NYSE	0.415	22.44	0.585	2.416	17.6%
MRTN	MARTEN TRANSPORT LTD	NASDAQ	0.412	18.84	1.310	2.900	46.2%
FPO	FIRST POTOMAC REALTY TRUST	NYSE	0.411	13.68	0.395	2.884	13.2%
IPAR	INTERMEDIATE PARFUMS INC	NASDAQ	0.410	13.58	0.751	1.540	48.9%
FRED	FREDS INC	NASDAQ	0.407	10.35	2.383	5.271	44.9%
MDCO	MEDICINES COMPANY	NASDAQ	0.407	7.70	4.613	10.934	42.7%
MIG	MEADOWBROOK INSURANCE GROUP	NYSE	0.405	7.08	0.212	1.697	12.7%
ANGO	ANGIODYNAMICS	NASDAQ	0.402	16.26	1.305	2.790	44.9%
PBH	PRESTIGE BRANDS HOLDINGS INC	NYSE	0.402	8.03	0.334	2.309	14.2%
BZ	BOISE INC	NYSE	0.401	4.75	0.224	2.256	13.0%
CDR	CEDAR SHOPPING CENTERS INC	NYSE	0.400	6.59	0.388	2.843	12.9%
APOG	APOGEE ENTERPRISES INC	NASDAQ	0.400	14.29	2.545	5.498	46.3%
MFB	MAIDENFORM BRANDS INC	NYSE	0.399	17.22	0.276	1.982	14.0%
EBF	ENNIS INC	NYSE	0.397	15.37	0.228	1.731	13.7%
FFIC	FLUSHING	NASDAQ	0.395	12.69	1.000	1.975	49.8%

CPSI	FINANCIAL CORP COMPUTER PROGRAMS & SYSTEMS INC	NASDAQ	0.394	35.94	1.946	4.217	47.5%
RIGL	RIGEL PHARMACEUTICALS	NASDAQ	0.392	7.55	4.175	9.221	43.0%
ABD	A C C O BRANDS	NYSE	0.391	7.17	0.406	2.973	13.3%
DK	DELEK U S HOLDINGS INC	NYSE	0.390	7.27	0.426	2.433	16.8%
CRVL	CORVEL CORP	NASDAQ	0.389	32.20	0.826	1.527	52.7%
CBZ	CBIZ INC	NYSE	0.389	6.23	0.284	2.235	13.4%
AZZ	A Z Z INC	NYSE	0.388	31.41	1.118	6.523	16.9%
CCO	CLEAR CHANNEL OUTDOOR HLDGS	NYSE	0.387	9.52	0.618	4.114	15.1%
BAS	BASIC ENERGY SERVICES INC	NYSE	0.385	9.45	1.075	5.781	17.4%
IMGN	IMMUNOGEN INC	NASDAQ	0.379	6.61	0.895	2.269	41.4%
MXWL	MAXWELL TECHNOLOGIES INC	NASDAQ	0.366	13.86	1.130	2.476	45.4%
CBEY	CBEYOND INC	NASDAQ	0.360	12.41	3.403	7.301	45.2%
ROCK	GIBRALTAR INDUSTRIES INC	NASDAQ	0.353	11.68	2.079	4.423	46.2%
NC	NACCO INDUSTRIES	NYSE	0.313	46.80	0.437	2.549	17.0%

B: Price Comparison Implementation Issues

Trade signing algorithms such as those presented in Section 2 conceptually require exact comparisons between prices, but directly implementing exact comparisons in commonly used statistical packages such as SAS can generate unexpected results due to floating point math issues. A problem arises when prices that appear to be integer multiples of the minimum tick size (or half of the minimum increment in the case of midpoints) are treated as floating point numbers. The same true price can be stored as slightly different floating point numbers on different trades or quotes. Floating point noise can also be introduced when a midpoint is calculated by averaging the bid and the ask. Exact comparisons will sometimes fail in these cases. This problem is well-known to computer scientists and is documented in the context of SAS applications in Go (2008) and Montgomery (2008), but we have not found any reference of it in the market microstructure literature.¹ Further, none of the publicly available trade signing sample code we are aware of robustly handles these comparisons.

As an illustration, consider the following excerpt from the WRDS sample code implementing the Lee and Ready (1991) algorithm:²

```
/* Lee and Ready Test */
/* Apply Quote Test first */
LeeReady=sign(Price-MidPoint&sec);
/* Then, Apply Tick Test */
if LeeReady=0 then LeeReady=TICK;
```

In the case of a trade price equal to the midpoint, when there is a floating point remainder in either the price or midpoint, the test `if LeeReady=0` will be evaluated as false, and the trade will be signed using the quote rule when it should have been signed using the tick rule. The quote

¹ Go (2008) constructs a simple example showing that SAS will find that 3×0.1 is not equal to 0.3.

² Retrieved from <http://wrds-web.wharton.upenn.edu/wrds/research/applications/intraday/index.cfm> on 8/21/2014.

rule will sign the trade based on the sign of the difference in floating point noise between the trade price and the midpoint. All other publicly available trade signing sample code that we are aware of is constructed similarly.

We recommend making all comparisons on rounded prices that allow for floating point noise of a magnitude smaller than half the minimum tick size. This is based on one of the solutions advocated in Go (2008), adjusted to take into account for trading increments possible in an electronic limit order book market. We use a noise band of \$0.005, forcing all prices to round to the nearest half-penny increment. If our sample had included sub-penny quotes (which we have removed by filtering out stock-days with trades occurring for \$1 or less), a finer granularity would be required. An excerpt from our SAS code applying this approach in a quote rule calculation is as follows:

```
/* quote rule - millisecond */
if price ~= . & mid_ms ~= . then do;
    if round(price,.005) > round(mid_ms,.005) then quote_rule_ms="B";
    else if round(price,.005) < round(mid_ms,.005) then
        quote_rule_ms="S";
    else quote_rule_ms="?";
end;
```

An analogous change to the WRDS sample code excerpt above would be:

```
/* Apply Quote Test first */
LeeReady=sign(Price-MidPoint&sec);
/* Then, Apply Tick Test */
if round(LeeReady,.001)=0 then LeeReady=TICK;
```

Table B.1 demonstrates the effects of exact comparisons that do not allow for floating point noise in our sample. We simply summarize the number and proportion of the trades in our sample where the trade price falls into the following ranges relative to the matched quotes: above the ask, at the ask, between the ask and the midpoint, at the midpoint, between the midpoint and

the bid, at the bid, and below the bid. The first two columns show the results when we naively do exact comparisons as in the WRDS sample code, the second two columns show the results when we implement our recommended procedure, and the last two columns show the differences. Examining the midpoint bucket, we see that the naïve exact approach incorrectly assigns over 1.8 million midpoint trades to other buckets, or almost 3% of our sample. These are trades that should be signed with the tick rule under the LR method that would be incorrectly signed with the quote rule instead. Similarly, the other two buckets that rely on an exact comparison to enter (at the ask and at the bid) together miss over 6.6 million trades under the naïve exact approach, or about 10.8% of the sample.

We do not claim that these effects are large enough to affect conclusions drawn in prior studies. Further, we cannot be sure that this affects datasets with different data formatting or storage strategies in the same way, and it is possible that other researchers (including the WRDS team) have handled the issues with exact comparisons in different ways. However, we suspect that other researchers unknowingly face this issue and it is therefore worth documenting.

References

- Go, I. C., 2008. Rounding in SAS: Preventing Numeric Representation Problems, SESUG Proceedings, PO-082.
- Montgomery, N., 2008. Floating Point Error: what, why, and how to!! PHUSE 2008 Paper CS08.
- Wharton Research Data System (WRDS), 2011. Identifying Buyer/Seller Initiated Trades using Lee and Ready (1991), WRDS Research Applications sample code.

Table B.1**Distribution of trade prices relative to quotes with two different comparison methods**

Numbers and proportions of sample trades classified into various ranges of trade prices relative to the quotes using exact comparisons and rounded comparisons that correct for floating point noise. Trade and quote data provided by NASDAQ. The sample period is the first full week of each quarter during 2008 and 2009, September 15, 2008 – September 19, 2008 (the week of Lehman’s failure), and February 22, 2010 – February 26, 2010. Trades are missing on October 10, 2008. Only trades between 9:30 am and 4:00:00 pm are used. Sample includes 61,271,087 trades.

Location	Exact Comparisons		Rounded Comparisons		Differences	
	N Trades	% Total	N Trades	% Total	N Trades	% Total
Above Ask	294,010	0.48	294,010	0.48	0	0
At Ask	21,628,271	35.3	24,912,182	40.66	-3,283,911	-5.36
Between Ask and Mid	6,212,379	10.14	2,383,568	3.89	3,828,811	6.25
At Mid	3,892,235	6.35	5,712,180	9.32	-1,819,945	-2.97
Between Mid and Bid	3,672,448	5.99	2,397,403	3.91	1,275,045	2.08
At Bid	21,942,441	35.81	25,272,336	41.25	-3,329,895	-5.44
Below Bid	3,629,303	5.92	299,408	0.49	3,329,895	5.43