## Intangible Capital, Volatility Shock, and the Value Premium

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<th>Journal:</th>
<th>The Financial Review</th>
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<td>Manuscript ID</td>
<td>FIRE-2017-07-107.R2</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Paper Submitted for Review</td>
</tr>
<tr>
<td>Keywords:</td>
<td>Intangible capital, economic uncertainty, value premium</td>
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</tbody>
</table>

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1. Introduction

Investment-based asset-pricing models, e.g., Zhang (2005), argue that the value premium exists in a production economy with rational expectations. Though this argument draws huge attention, the investment-based explanation begets a dilemma. On the one hand, value firms are riskier than glamour firms because highly asymmetric capital adjustment cost, together with counter-cyclical price of risk, leads physical capital to be harder to adjust especially in recessions. On the other hand, the same argument implies that assets in place are necessarily riskier than growth options in such an economy, completely opposite to the conventional wisdom in the literature.  

This paper proposes a remedy that reconciles the predicament. I postulate an economy with two types of assets in places and time-varying economic uncertainty. In particular, I extend the canonical, neoclassical growth model to include intangible capital and conditional uncertainty shock. I use the augmented investment-based asset-pricing model to conclude that complementarity in an intangible capital accumulation process together with time-varying volatility can generate a notably positive value spread in a production economy with rational expectations. The magnitude of the observed value premium in this new production economy with intangible capital is free from the abnormally asymmetric capital adjustment cost assumption that appears to create a quandary on the investment-based explanation.

Recent studies have found that the production side of the real economy appears to be heavily dependent on intangible capital. McGrattan and Prescott (2005) estimate that the value of intangible capital in U.S. corporations exceeds 60% of gross domestic product. Corrado, Hulten, and Sichel (2005) argue that the stock of intangible capital is as large as that of physical capital. Aside from the exact magnitude of the intangible capital stock, it is safe to say that intangible capital is a crucial component of the market price of a firm to the extent that the market price of the firm measures the value of all its different forms of productive capital (e.g., plants, structures,  

\[1\] Refer to Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), and Hillier, Grinblatt, and Titman (2011).

\[2\] Zhang (2005)'s economy is characterized by one type of capital and no time-variation in economic uncertainty.
know-how, employee expertise, organization capital, etc.). Data also suggest that U.S. firms have steadily increased the stock of intangible capital over the past 60 years, e.g., Hall (2001) and Falato, Kadyrzhanova, and Sim (2013).

Recent literature on fluctuating economic uncertainty emphasizes that the impact of temporary volatility shock appears to be salient in the production side of the real economy, e.g., Bloom (2009) and Arellano, Bai, and Kehoe (2012). In particular, Bloom (2009) shows that an increase in aggregate volatility is associated with an increase in the dispersion of firm profit growth, firm stock return, total factor productivity, and GDP forecast. Optimal investment decision under conditions of fluctuating economic uncertainty, e.g., Lucas and Prescott (1971) and Bloom, Bond, and Van Reenen (2007), is a key driver of the correlation reported in the literature. It turns out that accounting for the causal link to asset returns has nonetheless proven far from simple. On both the empirical side and the theoretical side, canonical investment-based asset-pricing models, e.g., Cochrane (1991) and Zhang (2005), have had difficulty reconciling the empirical phenomena that economic uncertainty fluctuation and joint uncertainty-cashflow relations determine stock return properties. Caballero (1991) also argues that the presence of asymmetric adjustment costs is not sufficient to render a negative relationship between investment and mean-preserving changes in uncertainty. A jointly dynamic uncertainty-investment structure has therefore become an important objective in the production-based asset-pricing literature.

The proposed jointly optimal investment dynamics for physical capital and intangible capital under time-varying volatility serve well to explain the cross-section of stock returns, providing a fresh insight into why physical capital-intensive value firms require more risk premium than intangible capital-intensive growth firms. It is costly to adjust the stock of physical capital at each plant. When the real economy is bad and economic uncertainty is high, firms sit tight until economic conditions become clearer to avoid reducing the stock of physical capital. Moreover,

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3 In linking the book-to-market ratio and intangible capital, I adopt standard empirical evidence as given. Lev and Sougiannis (1999) show that “Low BM [book-to-market] companies have a large R&D capital, while high BM companies have low R&D investment.” Ai and Kiku (2016) also point out that “high R&D spending and high Tobin’s Q, low leverage, and low dividend yields ... are characteristic of growth firms.”

4 See Bernanke (1983), McDonald and Siegel (1986), Eberly (1994), and Bloom (2009), for example.
idiosyncratic volatility is higher than aggregate volatility, e.g., Campbell, Lettau, Malkiel, and Xu (2001). Hence, a significant increase in idiosyncratic volatility at each level of disaggregation (i.e., from the macroeconomy to each plant) exacerbates the inaction on physical investment at each plant. On the contrary, it is not the case for intangible capital. The complementarity between already-acquired intangible capital and new intangible investment implies that firms have an incentive to sustain their intangible investment at a certain level in order to be productive. Consequentially, the real option effect is more salient for physical capital investment, leading physical-capital-intensive value firms to have cash flows that are more sensitive to the state of the real economy.

I incorporate this idea into the canonical, investment-based asset-pricing model by assuming the usual law of motion for physical capital stock but assuming that the stock of intangible capital evolves nonlinearly in a complementary Cobb-Douglas manner. The standard production-based asset-pricing framework deals with the evolution of intangible capital in the same way that physical capital develops over time. Griliches (1979), Hall and Hayashi (1989), and Klette (1996) point out that this commonly recognized accumulation process has unrealistic implications in two ways. First, if we assume the same law of motion for the two capital stocks, intangible capital is supposed to be symmetric in correspondence with physical capital. Second, already-acquired intangible capital and current intangible investment (e.g., research and development spending) are substitutes in the commonly assumed law of motion for intangible capital stock. Taken together, intangible investment is supposed to be lumpy and intermittent, comparable to physical investment, which is not the case in the data. Unlike the usual law of motion, I have chosen a nonlinear complementary knowledge production function. The new specification of intangible capital accumulation process incentivizes firms to persistently spend on intangible investment.

5Intangible capital is embodied into a firm in the form of R&D, organization capital (Eisfeldt and Papanikolaou, 2013), brand capital (Belo, Lin, and Vitorino, 2014b), customer capital (Gourio and Rudanko, 2014), etc. I find that R&D spending is less noisy than other measures of intangible investment. I thus use the term R&D as a concrete and representative example for intangible investment.

6Danthine and Jin (2007) also find that “the accumulation process for intangible capital differs significantly from the process by which physical capital accumulates.”

7A direct empirical test is presented in Section 2.
due to the complementarity between the existing stock of intangible capital and new intangible investment.

I derive the value premium and find that it is unconditionally positive *ex ante* in the model. Once installed, re-adjusting the stock of physical capital at a fire-sale price, as in Shleifer and Vishny (1992), is costly. Therefore, when the real economy is uncertain, firms take more precautions in physical investment to avoid selling off their tangible assets in the future, generating the real option effect. In contrast, current intangible investment complements the existing stock of intangible capital. This complementarity between current intangible investment and the stock of already-acquired intangible capital implies that the marginal product of intangible investment is decreasing in the amount of the current intangible investment project under standard assumptions. Firms thus have the incentive to stabilize intangible investments over time. As a result, physical investment responds to fluctuating economic uncertainty more negatively than intangible investment. Since stocks whose cash flows are more sensitive to economic conditions require higher expected returns, the value premium is significantly positive.

The value spread is not only reliably positive but also conditionally responsive to transient volatility shocks. Conditional upon a volatility shock in the economy, the expected return of value firms surges temporarily, implying that the realized value premium plummets (often to negative in the data). The volatility shock is short-lived, so the realized value premium tends to revert to the unconditional mean upon resolution of the volatility shock. This channel supports Chen, Petkova, and Zhang (2008) in explaining the puzzlingly low performance of value strategies during high volatility periods in the U.S.

This paper offers a new channel through which to understand the value premium from the production side of the real economy. First, this paper links the value premium to the optimal investment decision with two types of capital under fluctuating uncertainty. The new jointly optimal investment dynamics explored in this paper add to the literature of the driving forces of the value premium.\(^8\) Second, I exemplify a real economic mechanism of the characteristic-based

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\(^8\) Several explanations have been advanced to explain the value premium. For example, rational variation by Fama and French (1993), Fama and French (1996), and Zhang (2005), investor sentiment by Bondt and Thaler (1985) and
asset-pricing models, e.g., Berk et al. (1999) and Gomes et al. (2003). Book-to-market ratio is a proxy for the responsiveness of corporate activities to the state of the real economy and thus has a predictive power in explaining the cross-section of stock returns. Third, this paper proposes a unified framework to analyze the determinants of corporate investment and financing decisions (e.g., Graham and Harvey, 2001) such as financial flexibility and economic uncertainty. Those determinants are intuitively telling, but it is hard to quantify them in a rational expectations equilibrium. My model successfully captures the characteristics of corporate investment and financing decision.

The rest of the paper is organized as follows. Section 2 provides empirical evidence that motivates a distinctive accumulation process of intangible capital as opposed to that of physical capital. Section 3 extends the standard neoclassical growth model to include intangible capital and uncertainty fluctuation. Section 4 discusses how to compute the model. A useful model is supposed to explain the data; therefore Section 4 also summarizes a detailed empirical assessment of the model. Section 5 presents my findings on the value premium through the lens of the proposed investment-based asset-pricing model. Section 6 concludes the paper.

2. Intangible capital and economic uncertainty

The key margin of this paper is the difference in the responses of physical and intangible investment to economic uncertainty. As such, an empirical analysis of the impact of time-varying volatility on the production side of the real economy is useful as a precursor to the model introduced in Section 3. I first establish the link among intangible capital, economic uncertainty, and output. I then use the results to motivate a new intangible capital accumulation process. I also present the main results of this paper before detailing the model to enable readers to have a quick sense of the main results.

2.1 A motivating example

On October 17, 1989, an earthquake, officially named the 1989 Loma Prieta earthquake, rocked San Francisco. The earthquake was a transient uncertainty shock because “it [the Dow] came back and ended up the day about 18-1/2 points or about 1/2 percent lower: that would be true of the broader S&P 500 as well”. It turns out that responses to the earthquake were dichotomous between value firms and growth firms. Anecdotal evidence shows that value firms had more white knuckles after major events. For example, after the earthquake, General Motors idled its plants for several weeks whereas Silicon Valley firms, e.g., Cisco, decided to maintain what they call “arks” that store water, food, and emergency supplies for all employees on site to last up to three days. We observe a similar pattern after major economic events such as the Asian Financial Crisis, the 9/11 terrorist attack, etc.

[Insert Figure 1 Here]

Hall and Hayashi (1989), Klette (1996), and Doraszelski and Jaumandreu (2013) offer a theoretical explanation behind the above anecdote. Plants are substitutes whereas R&D engineers and scientists are complements. For simplicity, let us assume that the marginal benefit of a homogeneous factory or a scientist in Figure 1 above is one. Since plants are substitutes, the gross benefit of eight plants in Figure 1 is eight in both cases: 2+2+2+2 in Case A and 1+3+1+3 in Case B. Nonetheless, upon an uncertainty shock in the economy, firms prefer Case B since the real option value is bigger. By contrast, scientists are complements, e.g., in a Cobb-Douglas manner. The gross benefit of eight scientists is hence 16 (= 2 × 2 × 2 × 2) in Case C, but nine (= 1 × 3 × 1 × 3) in Case D. Even when the real economy is uncertain, firms thus have an incentive to maintain their intangible investment to maximize the gross benefit of knowledge capital. This explanation is consistent with the empirical fact that physical investment is lump-sum and intermittent, while R&D is highly persistent in the data.

As discussed above, the theoretical literature on the impact of economic uncertainty focuses

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9See Federal Open Market Committee Conference Call Minutes on October 18, 1989.
on the real option effect. For instance, firms have the option to open a gold mine or undertake a new investment project but do not need to exercise the option immediately. Once the option is exercised, opportunity cost to reverse the option exercise is large. Hence, the more uncertain the real economy is, the greater the incentive to keep the irreversible option and wait for more information. Physical investment decisions cannot be easily reversed due to adjustment costs, so it is better to pause temporarily when the economy turns bad. When the economy is highly uncertain, both productive firms and unproductive firms become less sensitive to the economic conditions. Productive firms are not expanding their capacity, and unproductive firms are not contracting enough to the optimal point. The caution induced by temporary economic uncertainty hinders the reallocation of resources across firms. As a result, high volatility makes firms temporarily intact in corporate activities; hence they wait to evaluate the situation until the economic smoke abates. However, scientists are different from plants. Both are capital goods, but they serve different purposes. First, more than half of R&D spending is the wages and salaries paid by firms to highly educated engineers and scientists whose efforts create firm-specific intangible capital (e.g., Hall, 2002). To the extent that the accumulated intangible capital is impossible to fully codify, firms have an incentive to smooth out their R&D spending in order to avoid frequently hiring and firing talented engineers and key persons. Second, the gestation period for R&D is longer than the conventional time-to-build period for ordinary physical investment. R&D is often carried out for a prolonged period of time to aim to yield long-term profit. As a result, physical investment is intermittent and lumpy whereas R&D spending is highly persistent.

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10 Real option effects arise when corporate decisions are not easily reversible. Thus, firms may continue to hire unskilled workers even when volatility is high because it is not costly to lay off part-time employees. Real option effects also depend on the degree of competition within an industry. For example, if firms are racing for a patent or a new product, the option value to defer spending will erode, e.g., Bloom (2014) and Patnaik (2015).

11 The first-order auto-correlation coefficient for yearly R&D spending of Compustat manufacturing firms in the United States for the period of 1981 to 2016 is 0.7693 while that of yearly physical investment is 0.4818. The second-order auto-correlation coefficients are 0.4818 for R&D and 0.3367 for physical investment.
2.2 Responsiveness to volatility

A more direct test is to look at the responses of physical investment and R&D to economic uncertainty shocks at the firm level. To confirm the prediction, I run a panel regression of physical investment and R&D on the Chicago Board Options Exchange (Cboe) S&P Volatility Index as well as a number of control variables. The data are quarterly and span from 1986 to 2016. I exclude those firms with less than a ten-year history to control for the entry-exit effect. Table 1 below reports the results. The responsiveness to market volatility is dichotomous between physical investment and R&D expenditure. Physical investment is significantly negatively associated with market volatility represented by the VXO index, whereas R&D expenditure is not responsive. The empirical evidence supports the view that there is a difference in the way the two types of capital are accumulated and exploited.

[Insert Table 1 Here]

The heterogeneous response of two capital assets to market volatility implies that the cross-sectional dispersion of investment (and thus output growth rates) is positively associated with market volatility. To further examine the impact of volatility shock on industrial production, I follow Sims, Stock, and Watson (1990) and run a monthly “short-run” orthogonal structural vector auto-regression (SVAR) in the order of the S&P 500 index, an indicator function for economic uncertainty shock, federal funds rates (FFR), average hourly earnings, Consumer Price Index (CPI), weekly hours, employment, and industrial production. The idea of this order is that volatility shock propagates into the economy through the stock market level first, prices (FFR, wages, and CPI), and then quantities (weekly hours, employment, and industrial production).
The indicator function for high volatility periods takes value 1 for each of the 17 shocks from 1972 to 2016 in Table 2. In principal, the periods of volatility shocks are chosen to be 1.65 standard deviation above the Hodrick-Prescott detrended mean with $\lambda = 129,600$ (Ravn and Uhlig (2002)) of the monthly realized volatility of the S&P 500 index. I then link the economic uncertainty indicator function to economic events such as war, disaster, or policy change as in Bloom (2009). I extend Bloom’s (2009) definition to include the Loma Prieta earthquake in October 1989, the liquidity shortfall in August 2007, the global financial crisis in October 2008, the Eurozone crisis in May 2010, and the European sovereign debt crisis in August 2011.

I choose six major manufacturing industries based on R&D intensity reported in Table 3 below: 1) Chemical (NAICS=325), 2) Computer (NAICS=334), 3) Medical Equipment (NAICS=3391), 4) Machinery (NAICS=333), 5) Electrical Equipment (NAICS=335), and 6) Plastics and Rubber Products (NAICS=326). I exclude highly concentrated industries measured by the Herfindahl-Hirschman index as firms in a highly concentrated industry have more freedom to cope with the state of the real economy, e.g., Patnaik (2015). Those six industries are chosen to be highly competitive, so that the industry structure does not play a role in explaining the link between economy-wide uncertainty shock and corporate decisions.

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16To investigate whether stock market volatility does jump, I use Barndorff-Nielsen and Shephard (2006)’s bipower variation test and find statistically significant evidence for volatility jumps. The bipower variation test rejects the null hypothesis that the realized monthly average volatility data are driven by a continuous semi-martingale process (H statistic = -1.4555 with p-value 0.07 and G statistic = -1.5366 with p-value 0.06). I further test the null of no-jumps using the VXO index from Cboe. The test results using the monthly VXO index reveals much stronger evidence for volatility jumps (H statistic = -6.0143 with p-value < 0.0001 and G statistic -8.7299 with p-value < 0.0001). Using the daily VXO data, the null hypothesis of no-jump is rejected at the 0.0001 significance level.


19North American Industrial Classification System (NAICS) codes do not seamlessly match with Standard Industrial Classification (SIC) codes. Hence, I tabulate R&D intensity for major manufacturing industries from 1999 when the U.S. Department of Commerce first reported industrial R&D as a percent of net sales by NAICS codes.

20The Herfindahl-Hirschman index is a measure of the degree of competition in the industry. The index is defined as $H = \sum_{i=1}^{N} s_i^2$ in which $N$ is the number of firms in the industry and $s_i$ is the market share of firm $i$ in percentage. The index ranges from 0 to 10,000, moving from a competitive market to a monopolistic market.
The percentage impact of volatility shock on industrial production, controlling for the S&P 500 index, FFR, wage, CPI, weekly hours, and employment, is plotted in Figure 2 below. I emphasize that the impact of stock market level on output is already controlled when backing out the impact of volatility shock on output. The impulse response function in Figure 2 shows that R&D-intensive industries in Panel A such as Chemical, Computer, and Medical Equipment cope well with a sudden volatility shock. Given a volatility shock, industrial production in those industries does not decline over the next 36 months. The ±1 standard error bound, denoted by dotted (+1 standard error) and dashed (−1 standard error) line, highlights that the results are significant at the 5% level. On the other hand, physical-capital-intensive industries in Panel B such as Machinery, Electrical Equipment, and Plastics and Rubber Products are highly responsive to volatility shocks. Industrial production falls sharply by 2-3% in three months, rebounding back to the original level in six months, and overshooting over the next two years. In sum, the data suggest that the output of R&D-intensive industries is less responsive to volatility shocks than that of physical-capital-intensive industries.

The results in Figure 2 indicate that the output of value stocks tend to be more responsive to volatility shocks than those of growth stocks in a competitive economy. The key difference between value and growth firms is their different responsiveness to the aggregate volatility shock. It is then clear that the cross-sectional dispersion of production is positively associated with economic uncertainty.

2.3 The mechanics of capital evolution dynamics

I formulate this idea by assuming the usual law of motion for the stock of physical capital:

\[ K_{i, t+1} = (1 - \delta_K)K_{i, t} + I_{i, t} \] (1)
but assuming that intangible capital is accumulated nonlinearly in a complementary Cobb-Douglas manner:

\[
G_{i,t+1} = \left( G_{i,t} \right)^{1 - \delta_G} \left( R_{i,t} \right)^{\delta_G}, \quad \delta_G \in (0, 1)
\]  

\[
R_{i,t} \geq 0
\]

where \( K_{i,t} \) is the stock of physical capital, \( \delta_K \) is the rate of physical capital depreciation, \( I_{i,t} \) denotes physical investment, \( G_{i,t} \) is the stock of intangible capital, \( \delta_G \) is the portion of R&D in knowledge production, and \( R_{i,t} \) indicates R&D expenditure. Also, R&D spending is assumed to have no resale value and accordingly cannot be reversed as in eq. (3).

Both eq.(1) and eq.(2) are linearly homogeneous, but the marginal product of physical investment is in sharp contrast to that of R&D. The marginal product of physical investment is unity in eq.(1) whereas the marginal return to R&D expenditure is decreasing in \( R_{i,t} \) in eq.(2) if \( \delta_G \in (0, 1) \):

\[
\frac{\partial K_{i,t+1}}{\partial I_{i,t}} = 1
\]  

\[
\frac{\partial^2 K_{i,t+1}}{\partial I_{i,t}^2} = 0
\]  

\[
\frac{\partial G_{i,t+1}}{\partial R_{i,t}} = \delta_G \left( G_{i,t} \right)^{1 - \delta_G} \left( R_{i,t} \right)^{\delta_G - 1}
\]  

\[
\frac{\partial^2 G_{i,t+1}}{\partial R_{i,t}^2} = (\delta_G - 1) \delta_G \left( G_{i,t} \right)^{1 - \delta_G} \left( R_{i,t} \right)^{\delta_G - 2} < 0.
\]

Consequently, eq.(1) and eq.(2) can successfully capture the observed empirical pattern that physical investment is lumpy and intermittent while R&D is spread out over time. Physical investment involves large and lump-sum expenditures that cannot be easily reversed. The unit marginal product of physical investment implies that firms time their business conditions for physical investment.

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\( ^{21} \)Hall and Hayashi (1989), Klette (1996), and Hall (2002) discuss how the stock of knowledge capital evolves over time. Doraszelski and Jaumandreu (2013) investigate the impact of R&D on productivity and provide empirical support for nonlinearity in the knowledge capital accumulation process.

\( ^{22} \)This term can be interpreted as the rate of intangible capital depreciation. Since \( \delta_G \) is defined in a non-arithmetic way, \( \delta_G \) is not directly comparable with \( \delta_K \).

\( ^{23} \)In computation, I use a normalizing constant \( \rho_0 \) to keep the stock of intangible capital from vanishing if the firm does not spend on R&D in a period: \( G_{i,t+1} = G_{i,t}^{1 - \delta_G} (\rho_0 + R_{i,t})^{\delta_G} \). The results are not affected by the choice of \( \rho_0 \).
vestment to avoid costly reversing the physical capital stock back to the previous level. On the contrary, lumpy R&D investment would decrease the marginal product of R&D due to eq.(6) and eq.(7), generating incentive to smooth out R&D spending over time.

2.4 The model’s solution

In this section, the main results of the model are presented before the model itself is described to enable readers to get the main results easily. I conduct comparative sensitivity analysis by varying 1) volatility shock, 2) adjustment cost, 3) price of risk, and 4) equity financing cost. This experiments help demonstrate which channels contribute more significantly to the value premium. Table 4 below reports the results from comparative analysis.

[Insert Table 4 Here]

I first shut the volatility shock channel down. If the conditional volatility for both productivity shock processes is constant, the value premium significantly shrinks down to 0.09% per month (equivalent to 1.12% per annum) in Table 4. Second, I apply the same law of motion for the two types of capital. The value premium decreases to 0.38% per month. Third, I set the adjustment cost function to be symmetric. The value spread is still notably positive and comparable to that from the data and the benchmark model. Fourth, the counter-cyclical price of risk does not significantly contribute to the value spread. The proposed investment-based model in this paper generates a significantly positive value spread (0.40% per month) even with symmetric adjustment cost and constant price of risk, respectively. Lastly, the existence of equity financing cost also marginally affects the value premium. Given the external financing cost structure where the fixed cost ($\gamma_0 = 0.0032$) is smaller than the variable cost ($\gamma_1 = 0.01$), value firms that issue equity in lump-sum bear more equity flotation cost.24 Taken together, the value spread is reliably positive and does not vary much in all of the specifications in Table 4 except for the case of constant conditional volatility, emphasizing the importance of time-varying volatility shock.25

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24The model’s calibration is discussed in Section 4.
25I tinker with a number of other specifications as well as those in Table 4. The results do not change.
3. The model

I take a canonical, neoclassical growth model and extend it in two ways. First, I incorporate intangible capital in the form of McGrattan and Prescott (2009)’s “technology capital.” Second, I allow a joint mix of aggregate productivity shock, idiosyncratic productivity shock, and economic uncertainty shock. The time-varying uncertainty shock interacts with aggregate productivity shock and idiosyncratic productivity shock and drives fluctuations in investment decisions. As is standard in the literature, I formulate an economy populated by a continuum of ex ante homogeneous consumers with unit mass and heterogeneous value-maximizing firms in a competitive market. Time is monthly from 0 to \( \infty \).

As the focus of this paper is on the production side of the real economy, I do not close the economy in general equilibrium. It is computationally more efficient to specify an exogenous stochastic discount factor.\(^{26}\) As long as I discipline the stochastic discount factor to match the rich aggregate consumption and risk-free rate dynamics observed in the data, this approach seems reasonable.

3.1 Firms

3.1.1 Profit

A firm consists of \( N \) identical production units. Each production unit produces a homogeneous final good that is either consumed or used for the production of another final goods. Intangible capital that firm \( i \) possesses can be costlessly exploited at all plants simultaneously. Following McGrattan and Prescott (2009) and McGrattan and Prescott (2010), I assume that the firm production function exhibits constant return-to-scale in intangible capital and composite production input. \( G_i \) units of intangible capital together with \( K_i \) units of physical capital pro-

\(^{26}\)Hence, aggregate consumption is exogenous whereas aggregate dividends are endogenous in the model. But both are affected by the same aggregate productivity shock process in the model. To validate the pricing kernel, I match the risk-free rate and the maximal Sharpe ratio induced by the assumed pricing kernel with those observed in the data.
duce:

\[ F_i = \tilde{A}(G_iN)^\phi(K_i^\alpha)^{1-\phi} \]  

(8)

in which \( \phi \) is the intangible capital share, \( \alpha \) is the physical capital portion in production input, and \( \tilde{A} \) represents a stochastic productivity shock process which will be defined later.

The demand for the final goods in the economy is iso-elastic with a constant elasticity \( \epsilon \) and a stochastic demand shifter \( \tilde{B} \):

\[ Q = \tilde{B}P^{-\epsilon}. \]  

(9)

This formulation leads to the following revenue function of firm \( i \):

\[
Y_i = P \times F_i \\
= \tilde{A}^{1-1/\epsilon}B^{1/\epsilon}(G_iN)^{\phi(1-1/\epsilon)}K_i^{\alpha(1-\phi)(1-1/\epsilon)}. 
\]  

(10)

Redefining \( a = \phi(1-1/\epsilon) \), \( b = (1-\phi)(1-1/\epsilon) \), and \( \tilde{A}^{1-1/\epsilon}B^{1/\epsilon} = A_i^{1-a-b} \), I have

\[
Y_i = A_i^{1-a-b}(G_iN)^a(K_i^\alpha)^b. 
\]

Here, I combine the productivity shock \( \tilde{A} \) and the stochastic demand shifter \( \tilde{B} \) into one term \( A_i \). Let \( f_K \geq 0 \) be the fixed cost of production (i.e., operating leverage). The net output produced by firm \( i \) is then:

\[
Y_i = A_i^{1-a-b}(G_iN)^a(K_i^\alpha)^b - f_K \]  

(11)

### 3.1.2 Technology

The economic condition \( A_{i,t} \) that firm \( i \) faces at time \( t \) is modeled as two different productivity shocks: aggregate productivity shock \( x_t \) and firm-level idiosyncratic productivity shock \( z_{i,t} \) with

\[
A_{i,t} = \exp(x_t + z_{i,t}). 
\]

As is standard in the literature, the aggregate productivity shock process \( x_t \) is assumed to
follow an exogenous stationary Markov process:

\[ x_{t+1} = \bar{x}(1 - \rho_x) + \rho_x x_t + \sigma_x^2 \epsilon_{t+1}^x, \quad \epsilon_{t+1}^x \sim N(0, 1) \] (12)

that aims to capture the total factor productivity at the macroeconomy, e.g., Cooley and Prescott (1995). The firm-level idiosyncratic productivity shock \( z_{i,t} \), which is crucial to generate a non-trivial cross-section of firms, follows a first-order auto-regressive process:

\[ z_{i,t+1} = \rho_z z_{i,t} + \sigma_z^i \epsilon_{t+1}^z \] (13)

where \( \epsilon_{t+1}^z \sim N(0, 1) \) is independent of each other for any \( i \) and \( t \).

### 3.1.3 Economic uncertainty

For simplicity, the stochastic processes for volatility are defined as a two-state Markov chain with transition probability \( \pi_{i,j} \):

\[
\begin{align*}
\sigma_x^t & \in \{\sigma_x^L, \sigma_x^H\} \\
\sigma_z^i & \in \{\sigma_z^L, \sigma_z^H\}
\end{align*}
\] (14) (15)

\[
P_r[\sigma_{t+1} = \sigma_j | \sigma_t = \sigma_i] = \pi_{i,j}
\] (16)

Eq.(14) and eq.(15) are parsimonious, but the process has been proven to be powerful enough to capture the data, e.g., Bloom (2009). Eq.(16) implies that the two stochastic conditional volatility processes are based on the same underlying Markov process. Hence, high-aggregate volatility states are associated with high idiosyncratic volatility states. This formulation enables the technology distribution at the firm level to jointly depend on conditional idiosyncratic volatility and, in turn, aggregate states of the economy, creating a mixed nonlinear structure for the technology distribution.\(^\text{27}\)

\(^{27}\)Finite-state Markov chains evolve over time in a discrete way, so that Markov regime-shifting models are more
3.1.4 Heterogeneous capital

As discussed in Section 2.3, there are two capital assets that evolve over time in a different way. As in eq.(1) and eq.(2), the laws of motion for the two types of capital are given as follows:

\[
K_{i,t+1} = (1 - \delta_K)K_{i,t} + I_{i,t}, \quad G_{i,t+1} = (G_{i,t})^{1-\delta_G} (R_{i,t})^{\delta_G}, \quad \delta_G \in (0, 1)
\]

\[R_{i,t} \geq 0\]

in which \(K_{i,t}\) is the physical capital stock, \(\delta_K\) is the rate of physical capital depreciation, \(I_{i,t}\) represents physical investment, \(G_{i,t}\) is the intangible capital stock, \(\delta_G\) is the portion of R&D in knowledge production, and \(R_{i,t}\) indicates R&D expenditure.

3.1.5 Adjustment cost

Adjusting the stock of capital goods is subject to nonconvex (or fixed) disruption cost (e.g., Cooper and Haltiwanger, 2006) and convex adjustment cost (e.g., Hall, 2001; Zhang, 2005). Following the literature, I specify a piece-wise quadratic adjustment cost function for physical investment:

\[
C^K_{i,t}(I_{i,t}, K_{i,t}) = \begin{cases} 
  b^+_{K}K_{i,t} + \frac{c^+_{K}}{2} \left( \frac{I_{i,t}}{K_{i,t}} \right)^2 K_{i,t} & \text{if } I_{i,t} > 0, \\
  0 & \text{if } I_{i,t} = 0, \\
  b^-_{K}K_{i,t} + \frac{c^-_{K}}{2} \left( \frac{I_{i,t}}{K_{i,t}} \right)^2 K_{i,t} & \text{if } I_{i,t} < 0.
\end{cases}
\]

in which \(b^-_{K} > b^+_{K} > 0\) and \(c^-_{K} > c^+_{K} > 0\) capture the fact that it is more costly to adjust tangible capital downward. Changing the stock of intangible capital through R&D investment also entails suitable for modeling discontinuous changes (i.e., jumps) in volatility. Generalized autoregressive conditional heteroskedasticity (GARCH) models are also simple enough, but it is impossible to decouple productivity and volatility using GARCH. To illustrate this, let us assume that productivity \((z_t)\) follows a first-order autoregressive process: \(z_{t+1} = \rho z_t + \sigma_t \epsilon_{t+1}\). \(\sigma_t\) is assumed to follow a GARCH process, say \(\sigma^2_{t+1} = \omega + \alpha (\sigma_t \epsilon_{t+1})^2 + \beta \sigma^2_t\). It is then impossible to disentangle a volatility shock from a level shock since both the level and the conditional volatility of idiosyncratic productivity are driven by one shock \((\epsilon_{t+1})\). For more details, refer to Fernández-Villaverde and Rubio-Ramírez (2010).
adjustment costs:

\[
C^G_{i,t}(R_{i,t}, G_{i,t}) = \begin{cases} 
0 & \text{if } R_{i,t} = 0, \\
 b_G G_{i,t} + \frac{c_G}{2} \left( \frac{R_{i,t}}{G_{i,t}} \right)^2 G_{i,t} & \text{if } R_{i,t} > 0.
\end{cases}
\] (18)

Here, I follow Hayashi (1982) and introduce a separate adjustment cost function. The incurred adjustment costs are directly cashed out from internal funds. The evolution of physical and intangible capital, i.e., eq. (1) and eq. (2), is therefore left intact.\(^{28}\)

The adjustment cost associated with changing the stock of intangible capital through R&D, together with the complementarity in the intangible capital accumulation process defined in eq.(2), may explain why differences in R&D intensity among firms persist over time. That is, even within the same industry, some firms do R&D to accumulate intangible capital while other firms do not spend on R&D at all. Because it is costly to initiate an R&D project, marginal firms do not spend on R&D. Once a firm kicks off an R&D project to boost its production technology, the intangible capital stock that the firm has acquired already through past R&D investments makes future R&D investments more productive due to the complementarity in eq.(2), leading the firm to continue to spend on R&D.

3.1.6 Financing

Firms can issue equity at any time. I denote the net payout to equity holders by \(E_{i,t}\). Equity flotation is costly (e.g., Hennessy and Whited (2007)) and can be described by

\[
\Gamma(E_{i,t}) = (\gamma_0 + \gamma_1 |E_{i,t}|) \mathbb{I}_{\{E_{i,t}<0\}}. 
\] (19)

Denoting distributions to equity holders by \(D_{i,t} = E_{i,t} - \Gamma(E_{i,t})\), corporate financing and

\(^{28}\)Another way to introduce adjustment cost can be found in Uzawa (1969) where adjustment costs are directly incorporated into the law of motion for capital.
investment decisions must satisfy the following resource constraint:

$$D_{i,t} = Y_{i,t} - I_{i,t} - C^K_{i,t} - R_{i,t} - C^G_{i,t}$$

(20)

### 3.1.7 Equity value maximization

Equity holders optimally choose physical investment $I_{i,t}$ and R&D investment $R_{i,t}$ to maximize the following Bellman equation:

$$V_{i,t}(K_{i,t}, G_{i,t}, \sigma_t, x_t, z_{i,t}) = \max \left[ 0, \max_{I_{i,t}, R_{i,t}} \left[ D_{i,t} + \mathbb{E}_t(M_{t+1}V_{i,t+1}) \right] \right]$$

(21)

subject to eq.(1), eq.(2), eq.(3), and eq.(20).

### 3.2 Households

Following Berk et al. (1999), Zhang (2005), and Kuehn and Schmid (2014), I do not close the model in general equilibrium. Rather, I take advantage of a parametric stochastic discount factor because the focus of this paper is on the production of the economy. As far as the assumed stochastic discount factor matches the aggregate dynamics observed in the data, this approach seems plausible.\(^{29}\)

In particular, the stochastic discount factor is given by:

$$\log(M_{t+1}) = \log \beta + \nu_t (x_t - x_{t+1})$$

(22)

$$\nu_t = \nu_0 + \nu_1 (x_t - \bar{x}), \quad \nu_1 < 0$$

(23)

in which $\beta$ is the representative consumer’s subjective discount factor, $x_t$ is the aggregate productivity shock process defined in eq.(12), and both $\nu_0$ and $\nu_1$ are constant.\(^{30}\) Since the stochastic

\(^{29}\)A detailed assessment of the assumed pricing kernel is discussed in Section 4.

\(^{30}\)The rationale behind eq.(22) is the following. The pricing kernel implied by a power-utility representative agent is given by:

$$\log(M_{t+1}) = \log \beta + RRA(c_t - c_{t+1})$$

(24)
discount factor is formulated as a function of aggregate productivity shock, eq.(12) and eq.(14) imply that the pricing kernel is also dependent on economic uncertainty shocks.\textsuperscript{31}

3.3 Rational expectations equilibrium

In a rational expectation equilibrium, the evolution $T_\mu$ of the cross-section of firms over time, together with the mapping $T_p$ from the aggregate state to the inter-temporal marginal rate of substitution of the representative household-owner $p$, characterize the economy from an individual firm’s perspective. Letters with a prime denote next period’s values.

\begin{align*}
\mu' &= T_\mu(\mu, \sigma, x) \\
p' &= T_p(\mu, \sigma, x)
\end{align*}

That is, given optimal policies, the cross-sectional distribution $\mu$ of firms at time $t$ satisfies:

\begin{equation}
\mu(\sigma', x', z') = P[\sigma_{t+1} = \sigma', x_{t+1} \leq x', z_{t+1} \leq z']
\end{equation}

4. Computing the model

A unique solution for continuous, concave, and bounded value function is guaranteed, thanks to Stokey and Lucas (1989). This solution means that numerical results will converge to the unique solution due to the fixed point theorem. Hence, I adopt numerical dynamic programming

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Example figure}
\end{figure}

where $c_t$ denotes the logarithm of aggregate consumption and $RRA$ is the coefficient of relative risk aversion. As in Zhang (2005), I tie $c_t$ with the aggregate state variable $x_t$ by assuming $c_t = a_c + b_c x_t$ for some constant $a_c$ and $b_c$, and defining $\nu_t = RRA \times b_c$. The result follows. The process $\nu_t = \nu_0 + \nu_1 (x_t - \bar{x})$ with $\nu_1 < 0$ is clearly decreasing in $(x_t - \bar{x})$, therefore, unlike the constant price of risk of power utility, the process $\nu_t$ with $\nu_1 < 0$ captures the fact that the price of risk is counter-cyclical.

\textsuperscript{31}On the consumption side, when economic uncertainty increases, people’s confidence in the future is shaken. For example, households may be thinking of buying a new car, but they could buy now or wait until next year. A household may be concerned about its income stability over the next few years due to high economic uncertainty. Rather than buying a new car immediately, it makes sense to defer spending on a car to avoid re-adjusting daily consumption items in the future. Another way to specify the pricing kernel is to use the fact that aggregate volatility shock carries a negative price of risk, e.g., Bansal, Kiku, Shaliastovich, and Yaron (2014). One can set the price of risk for aggregate volatility shock to be negative and let the model generate the cross-sectional variation in the loadings of volatility shock. See Koh (2017) for further details.
to approximate the competitive equilibrium.

I use iterative procedure to maximize the value function. I begin with an initial guess for the value function and solve for the firm’s optimization problem in eq.(21) on discrete state space.\textsuperscript{32} I iterate the procedure until the value function converges.

### 4.1 Approximating autoregressive processes

I follow Rouwenhorst (1995) to approximate the aggregate productivity process \( x_t \) defined in eq.(12).\textsuperscript{33} As the same procedure can be applied to the idiosyncratic productivity shock process, I focus on how to discretize the aggregate productivity process below.

I assume that \( x_t \) can take \( n_x \) values, say \( \{x_1, \ldots, x_{n_x}\} \) over the interval \([-\bar{\epsilon}, \bar{\epsilon}]\) with \( x_1 = -\bar{\epsilon} \) and \( x_{n_x} = \bar{\epsilon} \). The variance of \( x \) is then given by

\[
\text{Var}[x] = \frac{\bar{\epsilon}^2}{n_x - 1} \tag{28}
\]

For any \( n_x \geq 2 \), two parameters \( p, q \in (0, 1) \) govern the transition matrix of the \( n_x \)-state Markov chain. Moreover, it can be shown that the first-order auto-correlation is equal to \( p + q - 1 \). Hence, the auto-correlation of eq.(12) and its variance can be matched by setting

\[
\frac{\bar{\epsilon}^2}{n_x - 1} = \frac{\sigma_x^2}{1 - \rho_x^2} \tag{29}
\]

\[
p = q = \frac{1 + \rho_x}{2} \tag{30}
\]

\textsuperscript{32}I first confine the state space to a closed and bounded (hence, compact) set (e.g., \([0, \bar{K}]\)) to have a unique solution in the dynamic programming. The state space is discretized using the McGrattan et al. (1998) method.

\textsuperscript{33}Tauchen (1986), Tauchen and Hussey (1991), and Adda and Cooper (2003) are also available. Kopecky and Suen (2010) show that the Rouwenhorst (1995) method is more robust than other approximation methods when the Markov process is highly persistent.
For \( n_x = 2 \), the two-state Markov chain transition matrix is defined as:

\[
\Theta_2 = \begin{bmatrix}
p & 1-p \\
1-q & q
\end{bmatrix}
\]

Working forward recursively, the \( n_x \)-state Markov chain transition matrix for any \( n_x \geq 3 \) is defined as:

\[
\Theta_{n_x} = p \begin{bmatrix}
\Theta_{n_x-1} & 0 \\
0' & 0
\end{bmatrix} + (1-p) \begin{bmatrix}
0 & \Theta_{n_x-1} \\
0 & 0'
\end{bmatrix} + (1-q) \begin{bmatrix}
0 & 0' \\
0 & \Theta_{n_x-1}
\end{bmatrix} + q \begin{bmatrix}
0 & 0' \\
0 & \Theta_{n_x-1}
\end{bmatrix}
\]

in which 0 is a \((n_x - 1) \times 1\) column vector of zeros. Because the conditional probability is supposed to sum to one, the middle rows besides the top and bottom one are divided by two.

### 4.2 Calibration

The model requires me to calibrate 27 parameters: nine for business conditions, seven for production function, three for pricing kernel, six for investment adjustment costs, and two for external equity financing cost. I list all the parameters in Table 5 below.

[Insert Table 5 Here]

I begin with technology parameters on which we have strong prior beliefs. The aggregate state variable \( x_t \) follows a stationary auto-regressive process. The autoregressive coefficient of the process is assumed to be 0.9933, and the conditional volatility is set to be 0.0040 for low volatility state and 0.0080 for high volatility state, respectively. \( \rho_x = 0.9933 \) and \( \sigma^2_L = 0.0040 \) at the monthly frequency are consistent with \( 0.98 = (0.9933)^3 \) and \( 0.0070 = 0.0040 \times \sqrt{1 + (0.9933)^2 + (0.9933)^4} \) at the quarterly frequency in the real business cycle literature (e.g., Cooley and Prescott, 1995). Following Bloom (2009), I assume that conditional volatility becomes twice during periods of high volatility states, i.e., \( \sigma^2_H = 2 \times \sigma^2_L \). The long-run mean of the aggregate productivity process, \( \bar{x} = -4.4 \), is calibrated to match steady-state capital stock.
The parameters $\rho_z = 0.9800$ and $\sigma^2_L = 0.20$ denote the degree of persistence and cross-sectional dispersion in the idiosyncratic productivity process. I thus restrict these two parameters to be able to unconditionally match the cross-sectional dispersion reported in Gomes (2001), Campbell et al. (2001), and Pástor and Pietro (2003). Also, consistent with the real business cycle literature (e.g., Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry, 2012), I assume that micro uncertainty fluctuates more than aggregate uncertainty.

The calibrated idiosyncratic productivity shock is about 50 times as volatile as the aggregated productivity shock. Such a high idiosyncratic volatility is necessary to generate a nontrivial cross-section of firms. Nonetheless, I stress that the value of a firm is not very responsive to the idiosyncratic productivity shock process $z_t$ because $z_t$ affects only cash flow. On the contrary, the aggregate productivity shock process $x_t$ affects both cash flow and discount rate concurrently. When the economy is in recession, a firm’s cash flow is low, and the representative consumer’s marginal utility will be high. These two effects lead firm value to be more sensitive to the aggregate productivity shock.

Volatility shock occurs once every three years ($\pi_{L,H} = 1/36$) in the model, and its half life is assumed to be two months ($\pi_{H,H} = \sqrt{2}$). I borrow these numbers directly from Bloom (2009) who figures out 17 uncertainty shocks in 46 years. The assumed transition probability from low volatility state to high volatility state, $\pi_{L,H} = 1/36$, is also close to 0.0278 reported in Bloom et al. (2012).

The second set of parameters is related to the production side of the real economy. Hence, I base my calibration on the macro-finance literature. The share of physical capital ($\alpha$) in the production input is 0.70. I set the intangible capital share in output ($\phi$) to be 0.07 following McGrattan and Prescott (2009) and McGrattan and Prescott (2010). The constant demand elasticity is assumed to be four as in Bloom (2009). This means that markup is $1/3$.\(^{34}\)

The monthly rate of physical capital depreciation ($\delta_K$) is set to be 0.01 following Cooper and Haltiwanger (2006) and Zhang (2005), implying 12% per year as is usual in the literature. The

\[
\frac{\epsilon}{1 - \epsilon}\frac{dPQ}{dQ} = \left(1 - \frac{1}{\epsilon}\right)Q^{-1/\epsilon}B^{1/\epsilon}. \text{ Hence, } P = \frac{\epsilon}{1 - \epsilon}\frac{dPQ}{dQ}.
\]
share of current R&D expenditure in generating intangible capital is set at 0.05 at the monthly frequency, largely consistent with Hall, Jaffe, and Trajtenberg (2000) and Falato et al. (2013). The fixed cost of production $f = 0.4580$ is chosen to match the average book-to-market of the model with that observed in the data (e.g., Gomes, 2001). The number of plants per firm ($N$) is assumed to be 10. I also try $N = 5$ and $N = 20$. The results are not sensitive to the number of plants.

The pricing kernel is supposed to be volatile enough to match the observed Sharpe ratio. I therefore discipline the stochastic discount factor to match the Sharpe ratio, and the first and second moments of risk-free rate observed in the data. This step can be accomplished by setting $\beta = 0.9933, \nu_0 = 26$, and $\nu_1 = -250$. Table 6 compares model-generated key aggregate moments with those observed in Campbell and Cochrane (1999) and Lustig and Verdelhan (2012). The first and second moments of real interest rate generated under the benchmark calibration are 0.0237 and 0.0356, respectively. Campbell and Cochrane (1999) also report similar numbers using the postwar US data.

[Insert Table 6 Here]

Table 6 above also reports Sharpe ratios computed on the aggregate productivity shock space. The average Sharpe ratio is 0.4116 throughout the business cycle in the benchmark model. The variation in Sharpe ratios is substantial. Sharpe ratios range between 0.0292 and 0.8014 on the grid of the aggregate productivity shock. The model-generated average Sharpe ratio is 0.6334 in recessions and 0.2271 in booms. The numbers are largely consistent with Lustig and Verdelhan (2012) who report that the Sharpe ratio ranges from 0.14 to 0.85 and the average Sharpe ratio is 0.66 in contractions and 0.35 in expansions. Having pinned down a set of key aggregate moments using the benchmark calibration, I emphasize that the stochastic discount factor has no more room to match the cross-section of stock returns, which is the focus of this paper.

To calibrate the parameters of adjustment costs, I follow Cooper and Haltiwanger (2006), Zhang (2005), Bloom (2009), and Belo, Lin, and Bazdresch (2014a). There is no consistent calibration in the literature, but the key idea is that it is more costly to adjust the stock of capital
downward. This goal can be accomplished by assuming $b_K^+ << b_K^-$ and $c_K^+ << c_K^-$. Hence, I set $b_K^+, b_K^-, c_K^+, c_K^-$ to be 0.02, 0.03, 3, and 30, respectively. I set $b_G = 0.03$ and $c_G = 3$ because it is presumably the case that R&D projects are more costly to initiate but less costly to continue, compared to physical investment.

I am left with those parameters related to external equity issuance cost. Firms can raise fund by issuing new equity whenever the cash flow is short of required physical and intangible investment in return for equity flotation costs. Following Hennessy and Whited (2007) and Kuehn and Schmid (2014), I assume that equity issuance incurs both fixed cost ($\gamma_0 = 0.0032$) and variable cost ($\gamma_1 = 0.01$).

5. Quantitative implications

5.1 The value premium

I now investigate the cross-section of stock returns. To compare the benchmark model with the data, I first summarize the characteristics of book-to-market decile portfolios observed in the data from 1962 to 2016. Table 7 shows that the value premium is unconditionally positive (0.47% per month) in the data. Also, the alpha from the Capital Asset Pricing Model (CAPM) for the high-minus-low book-to-market portfolio is significantly positive (0.47% with $t = 2.60$).

[Insert Table 7 Here]

To see how the benchmark model matches the data, I simulate 100 artificial samples, each with 1,000 firms, from the benchmark model and compute means, standard deviations, alphas, and betas from the CAPM. For each simulation, I generate a path for conditional volatility, aggregate productivity shock, and idiosyncratic productivity shocks for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. When constructing the samples, I match each sample with Fama and French (1992) and Fama and French’s (1993) timing convention and run the CAPM regression. Panel B of Table 7 shows that the benchmark
model successfully generates a notably positive value spread. The model-generated value spread closely matches that of the data. The value premium in the benchmark model is 0.45% per month unconditionally, close to the observed value premium of 0.47% per month in the data.\footnote{Even though the benchmark model does a reasonable job in explaining the value spread, the model fails to capture the failure of the CAPM. The estimated CAPM alpha of the high-minus-low book-to-market portfolio is not significantly different from zero in Panel B. It is because the economy is modeled in a dynamic factor framework where exogenous state variables perfectly capture the model economy and the cross-section of firms generated by idiosyncratic shocks will be eventually integrated out. Given that I parameterize the benchmark model in a dynamic factor structure, this failure suggests that the pricing kernel should be more complex in the model.}

\section*{5.2 The impact of volatility shock on stock returns}

I define an indicator function for model-implied volatility shock to compare the empirical evidence discussed in Section 2 with the model-generated data. The stochastic processes for volatility are assumed to be a two-state Markov in eq.(14), eq.(15), and eq.(16). Hence, the model-implied indicator function for volatility shock equals one when the two-state Markov chain is in the high volatility state. I then run the following two regressions using simulated data to check the validity of the model.

\begin{align}
    r_{i,t} &= \alpha_i + \beta_i^V \mathbb{1}_{\text{volatility shock}} + \epsilon_{i,t} \\
    r_{i,t} &= \alpha_i + \beta_i^M MKT_t + \beta_i^V \mathbb{1}_{\text{volatility shock}} + \epsilon_{i,t}
\end{align}

where $r_{i,t}$ is the excess return of each of the book-to-market decile portfolios at time $t$, $\mathbb{1}_{\text{volatility shock}}$ is the volatility shock indicator function, and $MKT_t$ is the market excess return.

[Insert Table 8 Here]

Table 8 above summarizes the results from eq.(31) and eq.(32) using the model-generated data. The factor loading of the value-minus-growth portfolio on volatility shock is significantly negative: -3.59 with $t = -3.02$ in Panel A and -3.87 with $t = -6.95$ in Panel B.\footnote{The risk price of volatility shock is negative in the literature. See Bansal et al. (2014).} This result implies that, when there is a market volatility shock, value stocks suffer more than growth stocks.
because the output of physical-capital-intensive value firms are more responsive to the state of the economy. Taken together, the benchmark model works well to establish a causal link between volatility shock and the cross-section of stock returns.

5.3 Demystifying the book-to-market ratio

Although firm characteristics are not directly linked to the states of the real economy in eq.(12) and eq.(16), book-to-market ratio has a predictive power in explaining the cross-section of equity returns. To see this predictive power, let us consider the first-order condition of the value function posited by eq.(21) with respect to $I_t$:

$$-1 - \frac{\partial C^K_t}{\partial I_t} + \mathbb{E}_t \left[ M_{t,t+1} \frac{\partial V_{t+1}}{\partial I_t} \right] = 0 \quad (33)$$

The law of motion for $K_t$ implies that $\frac{\partial K_{t+1}}{\partial K_t} = 1$. Hence, we can replace $\frac{\partial V_{t+1}}{\partial H_t}$ with $\frac{\partial V_{t+1}}{\partial K_{t+1}}$ in eq.(33). Also, the gross investment return $R_{t,t+1}^I$ from $I_t$ is supposed to satisfy the following Euler equation:

$$\mathbb{E}_t [M_{t,t+1} R_{t,t+1}^I] = 1 \quad (34)$$

Combining eq.(33) and eq.(34) yields:

$$R_{t,t+1}^I = \frac{\frac{\partial V_{t+1}}{\partial K_{t+1}}}{1 + \frac{\partial C^K_t}{\partial I_t}} \quad (35)$$

where $\frac{\partial V_{t+1}}{\partial K_{t+1}}$ is the marginal benefit of physical capital and $1 + \frac{\partial C^K_t}{\partial I_t}$ is the marginal cost of investment. The marginal cost of investment is nothing but the marginal $q$. Putting it differently,

$$1 + \frac{\partial C^K_t}{\partial I_t} = q = \mathbb{E}_t [M_{t,t+1} \frac{\partial V_{t+1}}{\partial K_{t+1}}] \quad (36)$$

The construction of the marginal benefit of investment on the right side of eq.(36) has been the focus of the literature so far. However, whatever structure we explore, the left side of eq.(36) links
the market-to-book ratio to the assumed factor model. In other words,

\[
E[M_{t,t+1} \frac{K_t}{V_t} \frac{\partial V_{t+1}}{\partial K_{t+1}}] = 1 = E_t[M_{t,t+1} R_{t,t+1}^I]
\]

Taken together, book-to-market ratio captures the optimal investment decision under fluctuating economic uncertainty. This result is in line with Berk et al. (1999) and Gomes et al. (2003) who link the book-to-market ratio directly to the stock return.

6. Conclusion

I have developed a parsimonious investment-based asset-pricing model in a dynamic factor structure. By linking the cross-sectional dispersion of firms to the state of the real economy, I create a non-orthogonal space-time continuum of firms. Intangible capital accumulated from persistent R&D investment is central to explaining the dichotomous impact of fluctuating economic uncertainty on production between intangible capital-intensive growth firms and physical capital-intensive value firms. The key mechanism is that complementarity between past and current intangible investment incentivizes firms to continue to spend on intangible investment, leading glamour firms to be less sensitive to the state of the real economy.

The benchmark model puts forth the canonical, neoclassical production models in that my model works well to explain the optimal investment decision under time-varying economic uncertainty. The value spread arises naturally due to the difference in sensitivities to the state of the economy. Value firms are more exposed to volatility shock and thus require higher premium. Unlike other production models, the benchmark model generates significantly positive abnormal return on portfolios sorted on book-to-market ratio. Moreover, the factor loading of volatility shock on the return differential between high and low book-to-market portfolio is reliably negative, implying that value firms are more negatively affected by sudden volatility shocks than

\[37\] Marginal q is equal to average q when production function is constant return-to-scale, and adjustment cost function is linearly homogeneous. See Hayashi (1982) for more details.
growth firms. Finally, I also show how characteristic-based asset-pricing models suggested by Berk et al. (1999) and Gomes et al. (2003) can be interpreted in an investment-based asset-pricing framework.

Future research is certainly necessary. The mechanism explored in this paper is asset complementarity and uncertainty fluctuation. A natural extension of the model is to include a separate stochastic uncertainty process in the pricing kernel and link the process to the productivity processes endogenously. A study at a more disaggregated level (e.g., plant) is also called for.
Appendix 1: Data for vector auto-regression

I detail the data used for the structural vector auto-regression (SVAR) in Section 2. The SVAR model is in the order of the S&P 500 index from the Center for Research in Security Prices, an indicator function for volatility shocks defined in Table 2, effective federal funds rates (series ID: FEDFUNDS from the Federal Reserve Research Database), average hourly earnings (Series ID: CES3000000008 from the Bureau of Labor Statistics), consumer price index (Series ID: CUSR0000SA0 from the Bureau of Labor Statistics), weekly hours (Series ID: CES3000000007 from the Bureau of Labor Statistics), employment (Series ID: CES3000000001 from the Bureau of Labor Statistics), and industrial production (Series ID: IP.G325.S for Chemical, IP.G334.S for Computer, IP.N3391.S for Medical Equipment, IP.G333.S for Machinery, IP.G335.S for Electrical Equipment, and IP.G326.S for Plastics and Rubber Products from the Bureau of Labor Statistics). The data are monthly from 1972 to 2016. All variables are Hodrick-Prescott detrended with $\lambda = 129,600$ to remove a slow-moving cyclical component (e.g., Ravn and Uhlig, 2002).
References


Figure 1. Responsiveness to transient economic uncertainty shock. We assume that the marginal benefit of each plant or scientist is one, that factories are substitutes, and that scientists are complements in a Cobb-Douglas manner. The gross benefit of eight plants is eight in both Case A and Case B. The gross benefit of eight scientists is 16 ($= 2 \times 2 \times 2 \times 2$) in Case C, but nine ($= 1 \times 3 \times 1 \times 3$) in Case D.
Figure 2. Impulse response function. This figure plots the impulse response function from the structural vector auto-regression (SVAR) for six major manufacturing industries: 1) Chemical (NAICS=325), 2) Computer (NAICS=334), 3) Medical Equipment (NAICS=3391), 4) Machinery (NAICS=333), 5) Electrical Equipment (NAICS=335), and 6) Plastics and Rubber Products (NAICS=326). The SVAR is in the order of the S&P 500 index from the Center for Research in Security Prices, an indicator function for volatility shocks defined in Table 2, effective federal funds rates (series ID: FEDFUNDS from the Federal Reserve Research Database), average hourly earnings (Series ID: CES3000000008 from the Bureau of Labor Statistics), consumer price index (Series ID: CUSR0000SA0 from the Bureau of Labor Statistics), weekly hours (Series ID: CES3000000007 from the Bureau of Labor Statistics), employment (Series ID: CES3000000001 from the Bureau of Labor Statistics), and industrial production (Series ID: IP.G325.S for Chemical, IP.G334.S for Computer, IP.N3391.S for Medical Equipment, IP.G333.S for Machinery, IP.G335.S for Electrical Equipment, and IP.G326.S for Plastics and Rubber Products from the Bureau of Labor Statistics). The data are monthly from 1972 to 2016. All variables are Hodrick-Prescott detrended with $\lambda = 129,600$ to remove a slow-moving cyclical component. Dotted and dashed lines represent the $\pm 1$ standard error bound, respectively.
Table 1: Investment responsiveness to economic uncertainty. This table reports the fixed-effect ordinary least squares (OLS) regression results of physical investment and R&D expenditure on market volatility. Time is quarterly. All independent variables are lagged by one quarter to avoid potential endogeneity issues. The sample covers January 1986 to December 2016 because the VXO index became available in 1986. Heteroscedasticity-consistent test statistics are in square brackets. N denotes the number of observations. ***, **, and * represent significance at the 1%, 5%, and 10% levels, respectively.
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<th>Event</th>
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<td>Assassination of JFK</td>
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Table 2: Major volatility shocks since 1962. This table shows major volatility shocks since 1962. I detrend the monthly S&P 500 volatility data using the Hodrick-Prescott filter with $\lambda = 129,600$ to remove a slow-moving cyclical component. Those shocks are chosen to be 1.65 standard deviation above the Hodrick-Prescott detrended mean. If one shock lasted for several consecutive months, I chose the first month. OPEC represents the Organization of the Petroleum Exporting Countries.
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<td>3.2</td>
<td>2.9</td>
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Table 3: R&D intensity by industry from 1999 to 2007. HHI denotes the Herfindahl-Hirschman index for 50 largest firms as of 2007 reported by the Economic Census. NAICS is the North American Industry Classification System. D means the case that data are withheld to avoid disclosing operations of individual firms. The R&D intensity data are from the National Center for Science and Engineering Statistics. NAICS codes do not seamlessly match with Standard Industrial Classification (SIC) codes. Hence, I tabulate R&D intensity for major manufacturing industries from 1999 when the U.S. Department of Commerce first reported industrial R&D as a percent of net sales by NAICS codes.
### Monthly Excess Return of Portfolios Sorted on Book-to-Market Ratio

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<th>8</th>
<th>9</th>
<th>Value</th>
<th>V-G</th>
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<td>(1) Data</td>
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<td>0.54</td>
<td>0.58</td>
<td>0.57</td>
<td>0.55</td>
<td>0.61</td>
<td>0.67</td>
<td>0.70</td>
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<td>(2) Benchmark model</td>
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<td>0.71</td>
<td>0.72</td>
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<td>0.84</td>
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<td>(3) No volatility shock</td>
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<td>0.47</td>
<td>0.47</td>
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<td>0.49</td>
<td>0.50</td>
<td>0.52</td>
<td>0.53</td>
<td>0.57</td>
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<tr>
<td>(4) Symmetric law of motion</td>
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<td>0.65</td>
<td>0.65</td>
<td>0.66</td>
<td>0.68</td>
<td>0.71</td>
<td>0.76</td>
<td>0.80</td>
<td>1.03</td>
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<td>(5) Symmetric adj. cost</td>
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<td>0.76</td>
<td>0.79</td>
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<td>0.91</td>
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<td>(7) No financing cost</td>
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<td>0.70</td>
<td>0.70</td>
<td>0.72</td>
<td>0.73</td>
<td>0.73</td>
<td>0.75</td>
<td>0.81</td>
<td>0.86</td>
<td>1.09</td>
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Table 4: Sensitivity analysis. This table reports the results from comparative sensitivity analysis by varying volatility shock, law of motion, adjustment cost, price of risk, and equity financing cost. The numbers are monthly average excess return (%) for each of the book-to-market portfolios.
<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<td>$\sigma_{\bar{L}, \bar{H}}$</td>
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<td>conditional volatility of the aggregate productivity process</td>
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<td>$\bar{x}$</td>
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<td>constant term of the aggregate productivity process</td>
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<td>$\sigma_{\bar{z}, \bar{H}}$</td>
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<td>one volatility shock per three years</td>
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<td>$\pi_{H,H}$</td>
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<td>two-month half life</td>
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<td>$\alpha$</td>
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<td>$N$</td>
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<td>the number of plants per firm</td>
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Table 5: Benchmark calibration. The model is required to calibrate 27 parameters: nine for economic environments, seven for production function, three for pricing kernel, six for investment adjustment costs, and two for external equity financing cost. Economic environments parameters are chosen to match Cooley and Prescott (1995). Production function parameters are set to match Zhang (2005) and McGrattan and Prescott (2009). I discipline the pricing kernel parameters to match the risk-free rate and the Hansen-Jagannathan bound, so that there is no more degree of freedom for the pricing kernel to capture the cross-section of firms, which is the focus of this paper. Adjustment cost parameters are from Cooper and Haltiwanger (2006) and Bloom (2009). Finally, external equity financing cost parameters are from Hennessy and Whited (2007) and Kuehn and Schmid (2014).
Table 6: Key aggregate moments. This table compares model-generated key aggregate moments with those observed in the data. The data are from Campbell and Cochrane (1999) and Lustig and Verdelhan (2012). I generate a path for eq.(12) for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. I then compute the mean and standard deviation of real interest rate. To compute the Hansen-Jagannathan bound, I discretize the aggregate productivity shock process using the Rouwenhorst (1995) method into 15 discrete intervals. Expansions are from the median to the largest $x_t$ over the discretized intervals. Recessions make up the remaining values.

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</table>
Table 7: Properties of portfolios sorted on book-to-market. I simulate 100 artificial samples, each with 1,000 firms, from the model. For each simulation, I generate a path for conditional volatility, aggregate productivity shock, and idiosyncratic productivity shocks for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. When constructing the samples, I match each sample with Fama and French (1992) and Fama and French’s (1993) timing convention and run the regression. Panel A summarizes the results from the Capital Asset Pricing Model using the real data from 1962 to 2016. Panel B reports the results from the benchmark model of this paper. Mean and SD are the mean and standard deviation of monthly excess returns in each portfolio. \( \alpha \) and \( \beta \) are the CAPM alpha and beta. \( t_\alpha \) and \( t_\beta \) are the Newey-West t-statistics for the CAPM alpha and beta.
Table 8: Volatility shock and value premium from the benchmark model. This table reports the results from the regression of 10 portfolio returns sorted on book-to-market ratio on the market excess return and the volatility shock indicator function using simulated data. Panel A reports the baseline regression: $r_{i,t} = \alpha_i + \beta_i^{V} \mathbb{1}_{\text{volatility shock}} + \epsilon_{i,t}$. Panel B shows the same regression controlling for the excess market return: $r_{i,t} = \alpha_i + \beta_i^{MKT} \text{MKT}_t + \beta_i^{V} \mathbb{1}_{\text{volatility shock}} + \epsilon_{i,t}$. The indicator function for volatility shock takes value 1 when the two-state Markov chain defined in eq.(14) and eq.(15) is in the high volatility state. I simulate 100 artificial samples, each with 1,000 firms, from the model. For each simulation, I generate a path for conditional volatility, aggregate productivity shock, and idiosyncratic productivity shocks for 200 years at the monthly frequency and remove the first 100 years to minimize the impact of initial conditions. When constructing the samples, I match each sample with Fama and French (1992) and Fama and French’s (1993) timing convention and run the regression. Newey-West t-statistic is reported in square bracket.